Hicksian Demands and Expenditure Function for Perfect Complements and Perfect Substitutes

Find the Hicksian demands and expenditure functions for the following utility functions:

1. $$U = \min\{\alpha x_1, \beta x_2\}$$

2. $$U = \alpha x_1 + \beta x_2$$

Subject to:

$$M = p_1 x_1 + p_2 x_2$$

And $\alpha, \beta > 0$

Solutions

1. For the individual to not waste money unnecessarily, it must be the case that:

$$\alpha x_1 = \beta x_2$$

From this, we obtain the Marshallian demands:

$$x_1^m = \frac{M\beta}{p_1\beta + p_2\alpha}$$

$$x_2^m = \frac{M\alpha}{p_1\beta + p_2\alpha}$$

With this, we get the indirect utility function:

$$V = \frac{\alpha\beta M}{\beta p_1 + \alpha p_2}$$

To obtain the expenditure function, we invert the function:

$$\bar{u} = \frac{\alpha\beta M}{\beta p_1 + \alpha p_2}$$

$$E = \bar{u}\frac{\beta p_1 + \alpha p_2}{\alpha\beta}$$

And now we derive it to obtain the compensated/Hicksian demands (Shephard's lemma)

$$x_1^h = \frac{\partial E}{\partial p_1} = \frac{\bar{u}}{\alpha}$$

$$x_2^h = \frac{\partial E}{\partial p_2} = \frac{\bar{u}}{\beta}$$

2. We assume without loss of generality that $\frac{p_1}{p_2} > \frac{\alpha}{\beta}$, therefore the Marshallian demands for the function of perfect substitutes are:

$$x_1^m = \frac{M}{p_1}$$

$$x_2^m = 0$$

The indirect utility function is:

$$V = \alpha\frac{M}{p_1}$$

We invert to find the expenditure function:

$$E = \frac{\bar{u}p_1}{\alpha}$$

We derive to find the compensated/Hicksian demands (Shephard's lemma)

$$\frac{\partial E}{\partial p_1} = x_1^h = \frac{\bar{u}}{\alpha}$$

$$\frac{\partial E}{\partial p_2} = x_2^h = 0$$

To obtain Hicksian demands in the case where $\frac{p_1}{p_2} < \frac{\alpha}{\beta}$, simply replace subscript 1 with subscript 2. And in the case where $\frac{p_1}{p_2} = \frac{\alpha}{\beta}$, there are infinite possible solutions including the endpoints for obtaining Marshallian demands, which leads to infinite solutions for Hicksian demands.